VICTORIA SHANGHAI ACADEMY

Part 1

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a) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^2$

$= \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$= \begin{pmatrix} 2\times2+1\times1 & 2\times1+2\times1 \\ 1\times2+2\times1 & 1\times1+2\times2 \end{pmatrix}$

$= \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

$L^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$ //

b) $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}^2$

$= \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 3\times3+2\times2 & 3\times2+2\times3 \\ 2\times3+3\times2 & 2\times2+3\times3 \end{pmatrix}$

$= \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$

$M^2 = \begin{pmatrix} 13 & 12 \\ 12 & 13 \end{pmatrix}$

c) $\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}^2$

$= \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 4\times4+3\times3 & 4\times3+3\times4 \\ 3\times4+4\times3 & 3\times3+4\times4 \end{pmatrix}$

$= \begin{pmatrix} 25 & 24 \\ 24 & 25 \end{pmatrix}$

$N^2 = \begin{pmatrix} 25 & 24 \\ 24 & 25 \end{pmatrix}$ //

d) $\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}^2$

$\begin{pmatrix} (a+1) & (a) \\ a & (a+1) \end{pmatrix}\begin{pmatrix} a+1 & a \\ a & a+1 \end{pmatrix}$

$= \begin{pmatrix} (a+1)(a+1)+(a)(a) & a\times(a+1)+(a\times(a+1)) \\ a\times(a+1)+(a+1)\times a & a\times a+(a+1)(a+1) \end{pmatrix}$

$= \begin{pmatrix} a^2+2a+1 & 2a^2+2a \\ 2a^2+2a & a^2+2a+1 \end{pmatrix}$

$A^2 = \begin{pmatrix} a^2+2a+1 & 2a^2+2a \\ 2a^2+2a & a^2+2a+1 \end{pmatrix}$ //

e) Xavi is wrong.

f)

g) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} 2\times3+2\times1 & 2\times2+3\times1 \\ 1\times3+2\times2 & 1\times2+2\times3 \end{pmatrix}$

$= \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix}$

$LM = \begin{pmatrix} 8 & 7 \\ 7 & 8 \end{pmatrix}$ //

h) $\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 3\times4+2\times3 & 3\times3+4\times2 \\ 2\times4+3\times3 & 3\times2+3\times4 \end{pmatrix}$

$= \begin{pmatrix} 18 & 14 \\ 17 & 18 \end{pmatrix}$

$MN = \begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix}$ //

i) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

$= \begin{pmatrix} 2\times4+1\times3 & 2\times3+1\times4 \\ 1\times4+2\times3 & 1\times3+2\times4 \end{pmatrix}$

$= \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix}$

$LN = \begin{pmatrix} 11 & 10 \\ 10 & 11 \end{pmatrix}$ //

let a be 2, b be 3, c be 4

$\begin{pmatrix} 2+1 & 2 \\ 2 & 2+1 \end{pmatrix}\begin{pmatrix} 3+1 & 3 \\ 3 & 3+1 \end{pmatrix}$

$=\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$

j) $=\begin{pmatrix} 3\times4+3\times2 & 3\times3+2\times4 \\ 2\times4+3\times3 & 2\times3+4\times3 \end{pmatrix}$

$=\begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix}$

$\begin{pmatrix} 18 & 17 \\ 17 & 18 \end{pmatrix} \neq \begin{pmatrix} 4+1 & 4 \\ 4 & 4+1 \end{pmatrix}$

so $AB \neq \begin{pmatrix} c+1 & c \\ c & c+1 \end{pmatrix}$

k) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^3$

$=\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$=\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$=\begin{pmatrix} 5\times2+4\times1 & 5\times1+4\times2 \\ 4\times2+5\times1 & 4\times1+5\times2 \end{pmatrix}$

$=\begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}$

l) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^4$

$=\begin{pmatrix} 14 & 13 \\ 13 & 14 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$=\begin{pmatrix} 14\times2+13\times1 & 14\times1+13\times2 \\ 13\times2+14\times1 & 13\times1+14\times2 \end{pmatrix}$

$=\begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}$

m) $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^5$

$=\begin{pmatrix} 41 & 40 \\ 40 & 41 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$

$=\begin{pmatrix} 41\times2+40\times1 & 41\times1+40\times2 \\ 40\times2+41\times1 & 40\times1+41\times2 \end{pmatrix}$

$=\begin{pmatrix} 122 & 121 \\ 121 & 122 \end{pmatrix}$

n) $\cancel{\begin{pmatrix} a & b \\ b & a \end{pmatrix}^2}$

$\cancel{=\begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix}}$

$\cancel{=\begin{pmatrix} a^2+b^2 & ab+ab \\ ab+ab & a^2+b^2 \end{pmatrix}}$

$\cancel{=\begin{pmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{pmatrix}}$

$\cancel{\begin{pmatrix} a & b \\ b & a \end{pmatrix}^3}$

$\cancel{=\begin{pmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix}}$

$\cancel{=(}$

From the answers above, I can see that whenever $\begin{pmatrix} a & b \\ b & a \end{pmatrix}^2$ or $\begin{pmatrix} a & b \\ b & a \end{pmatrix}^3$ or $\begin{pmatrix} a & b \\ b & a \end{pmatrix}^4$ or $\begin{pmatrix} a & b \\ b & a \end{pmatrix}^5$ they are all equal the same $=\begin{pmatrix} a & b \\ b & a \end{pmatrix}$

$\begin{pmatrix} a & b \\ b & a \end{pmatrix}^2$

$=\begin{pmatrix} a & b \\ b & a \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix}$

$=\begin{pmatrix} a^2+b^2 & ab+ab \\ ab+ab & a^2+b^2 \end{pmatrix}$

$=\begin{pmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{pmatrix}$

$\cancel{\begin{pmatrix} a & b \\ b & a \end{pmatrix}^3}$

$\cancel{=\begin{pmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix}}$

$=$

~~The pattern is~~

$\cancel{\begin{pmatrix} a+1 & b+1 \\ b+1 & a+1 \end{pmatrix}}$

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Part 2

12/1/2012

a) let T be $\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$

$= \begin{pmatrix} 0\times1+(-1)\times2 & 0\times1+5\times(-1) & 0\times3+(4\times-1) \\ 1\times-1+0\times2 & 1\times-1+0\times5 & 3\times-1+0\times4 \end{pmatrix}$

$= \begin{pmatrix} -2 & -5 & -4 \\ -1 & -1 & -3 \end{pmatrix}$

b) I find that after multipling $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ the matrix will all become negative and they change their places upside down ~~this is a reflection.~~ ~~MAT repersent the it is reflected~~.
it is reflected ~~and~~ in the negative side.

~~c) Reflection~~

d) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^2\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1\times1+0\times2 & 1\times1+0\times5 & 1\times3+0\times4 \\ 0\times1+1\times2 & 0\times1+1\times5 & 0\times3+1\times4 \end{pmatrix}$

$= \begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$

e) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^3\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$

$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$

$= \begin{pmatrix} -2 & -5 & -4 \\ -1 & -1 & -3 \end{pmatrix}$

f) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^{-1}$

$= \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\dfrac{1}{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}}$

$= \dfrac{1}{\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

g) When the power of the matrix is an even number, the matrix will become a positive answer and remain unchange

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^2\begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$

$= \begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$

but if the power of the matrix is an odd number, the matrix will become a negative number and the answer will turn upside down.

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example of a power is an odd no.

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^3 \begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$$

$$= \begin{pmatrix} -b & -d & -f \\ -a & -c & -e \end{pmatrix}_{\!/\!/}$$

by look at the power ($M^{89998}T$)., if they power is an even number, you will know it remain unchange. You can also use a calculater to calculate the answer.

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}^{8998} \begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 3 \\ 2 & 5 & 4 \end{pmatrix}_{\!/\!/}$$

I think the one that look at the power is an odd or even number is the most appropriate because it is faster.

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c)